Harmonic analysis of irradiation asymmetry for cylindrical implosions driven by high-frequency rotating ion beams

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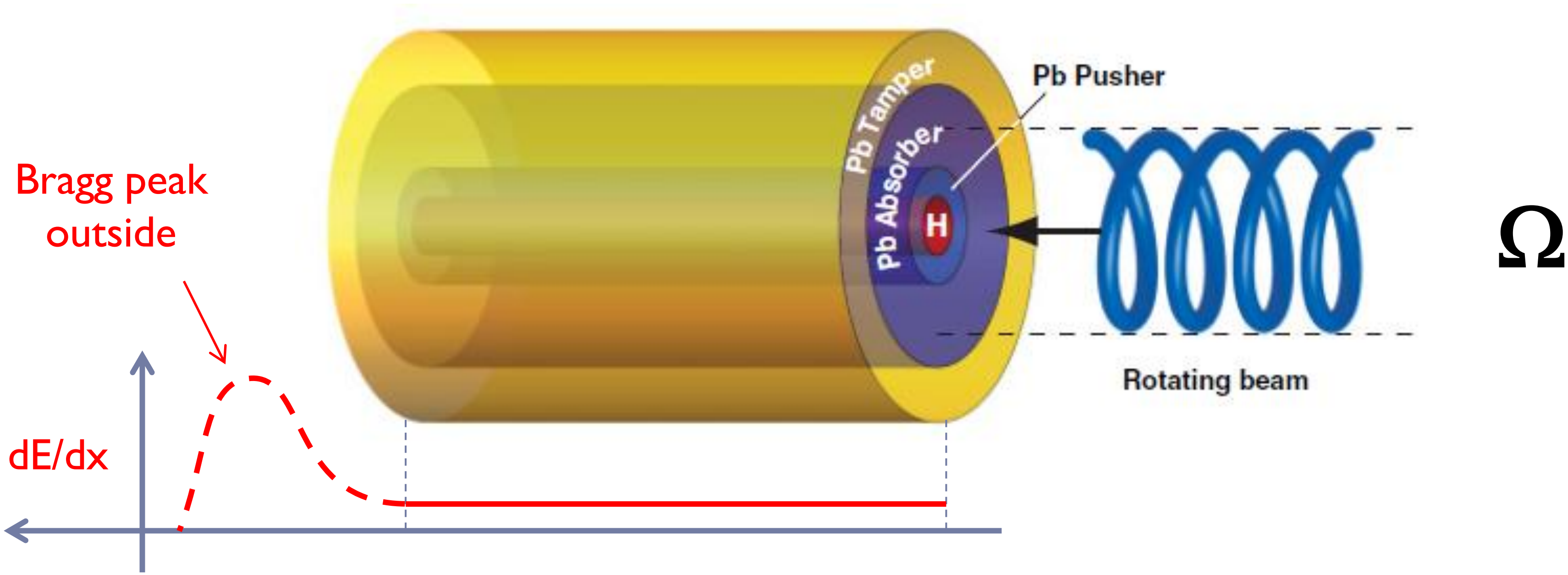
The LAPLAS Experiment

- "Laboratory of Planetary Sciences" (LAPLAS) at GSI
- HEDM, Metallic Hydrogen…

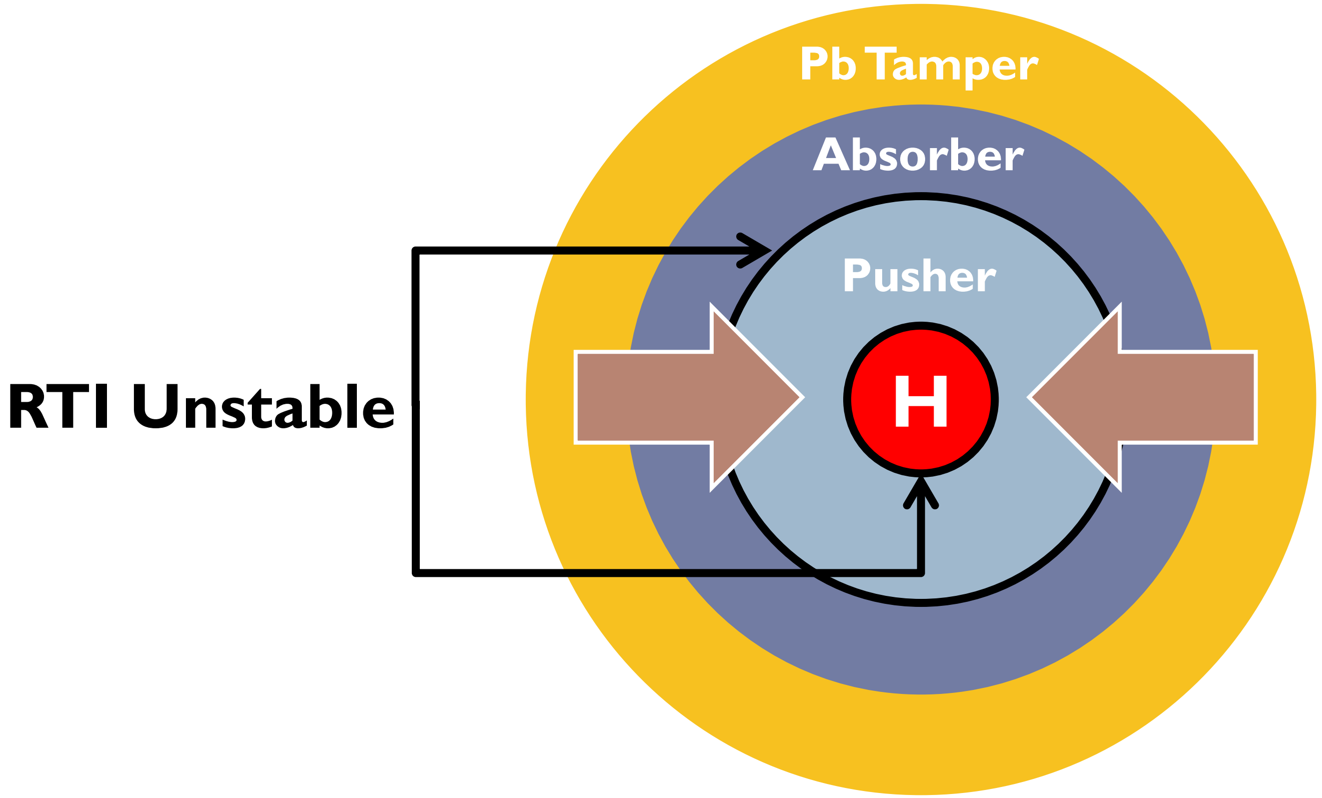


- **Rayleigh-Taylor** instability during implosion?

- Tahir et al., Phys. Rev. E **61, 1975 (2000).**

Rayleigh-Taylor instability

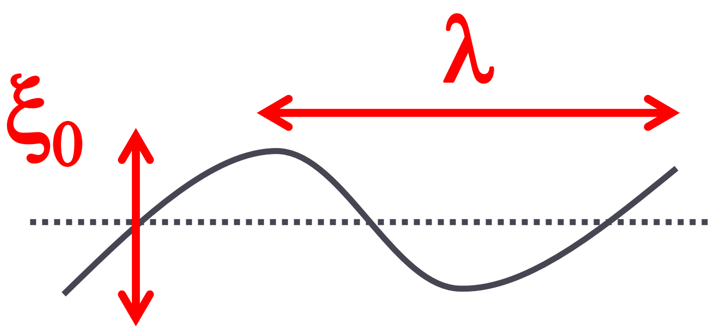
- Unstable interfaces during the implosion
- RTI in **elastic-plastic solids** (RTI-EPS, Piriz 2009)



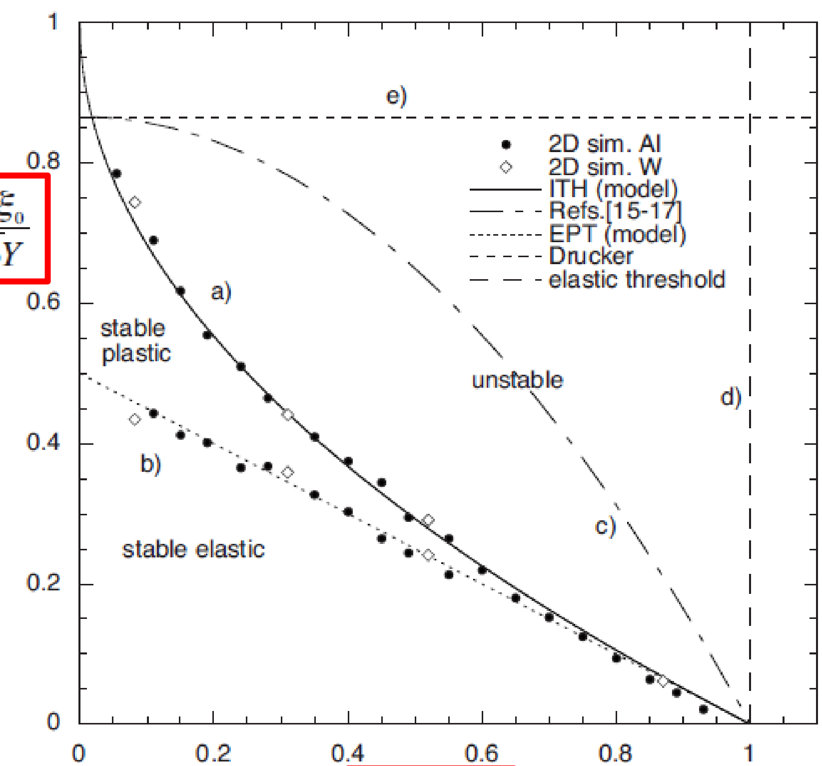
- Piriz *et al.*, Phys. Rev. E **80**, 046305 (2009)

Irradiation asymmetries

- Beam **time profile** → irradiation asymmetries.
- Planar RTI-EPS in terms of λ, ξ (Piriz 2009)



$$\hat{\xi} = \frac{\rho g \xi_0}{\sqrt{3Y}}$$

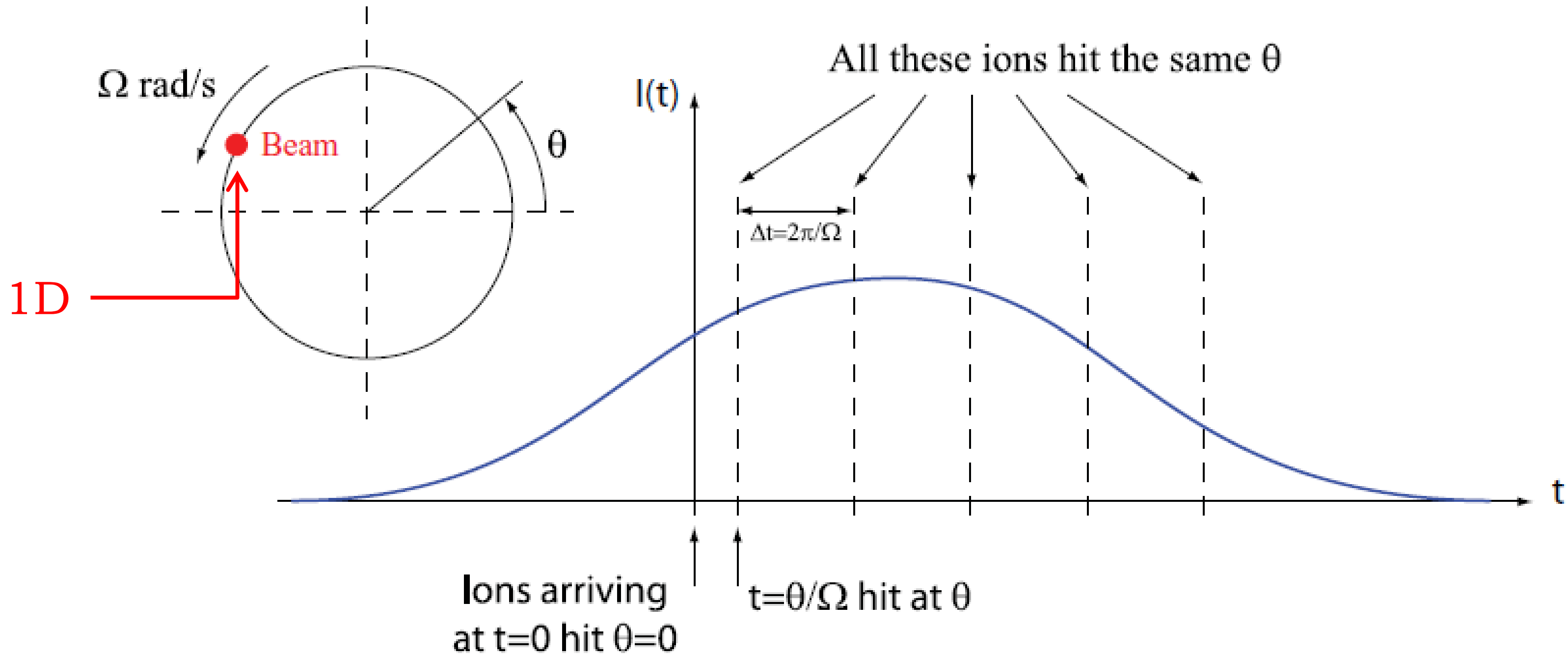


$$\hat{\lambda} = \frac{\rho g \lambda}{4\pi G}$$

- **Which (λ, ξ) are produced?**

Piriz *et al.*, Phys. Rev. E **80**, 046305 (2009)

Irradiation Fourier spectrum – 1D beam



$$dN(\theta) = \sum_{l=-\infty}^{\infty} I\left(\frac{\theta}{\Omega} + \frac{2l\pi}{\Omega}\right)\frac{d\theta}{\Omega} \quad \Rightarrow \frac{dN(\theta)}{d\theta} = \frac{1}{\Omega}\sum_{l=-\infty}^{\infty} I\left(\frac{\theta + 2l\pi}{\Omega}\right) \equiv \rho(\theta)$$

- Bret *et al.*, Phys. Rev. E **85**, 036402 (2012)

Irradiation Fourier spectrum, 1D beam

- Now, Fourier transform $\rho(\theta)$:

$$\widehat{\rho}(s) = \int_{-\infty}^{\infty} \rho(\theta) e^{is\theta} d\theta = \frac{1}{\Omega} \int_{-\infty}^{\infty} \sum_{l=-\infty}^{\infty} I\left(\frac{\theta + 2l\pi}{\Omega}\right) e^{is\theta} d\theta$$

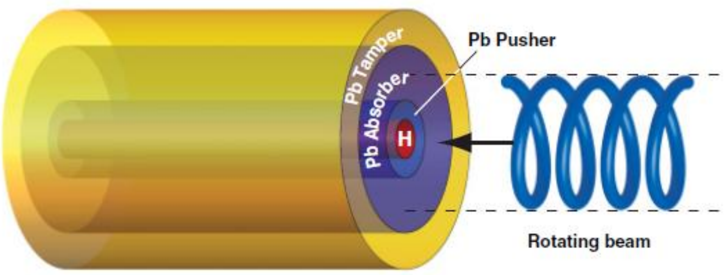
$$\ldots = \underbrace{\left(\int_{-\infty}^{\infty} I(u) e^{is\Omega u} du\right)}_{\text{Fourier Transform of beam profile, } \hat{I}(s\Omega)} \underbrace{\sum_{l=-\infty}^{\infty} e^{-2il\pi s}}_{\text{“Dirac's comb”} = \sum_{l=-\infty}^{\infty} \delta(s-l)}$$

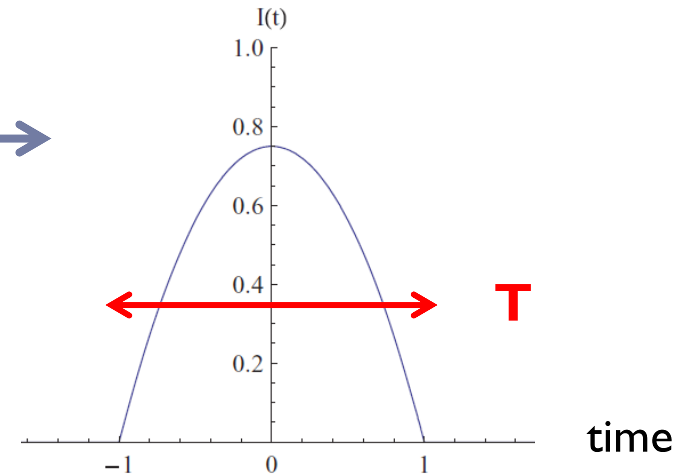
$$\boxed{\widehat{\rho}(l) = \widehat{I}(l\Omega) + \widehat{I}(-l\Omega), \quad \forall \;\; l \in \mathbb{N}}$$

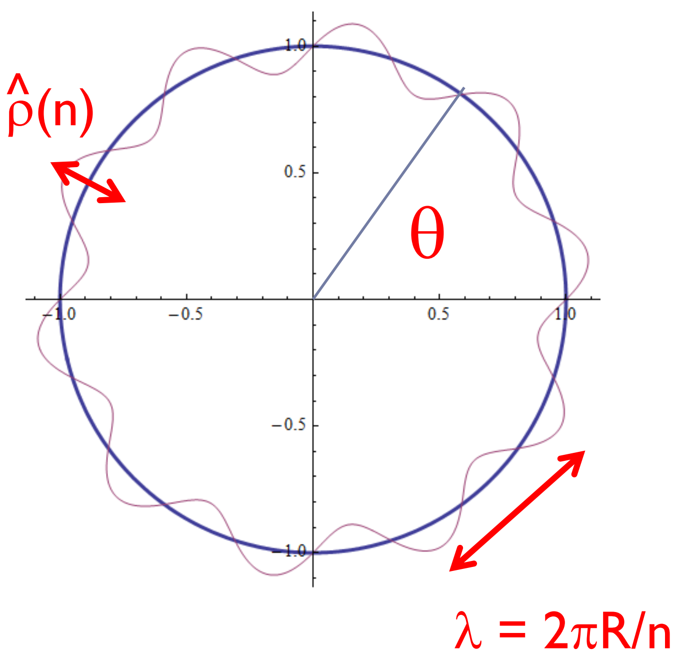
Harmonic amplitudes of irradiation, from Fourier Transform of beam time profile

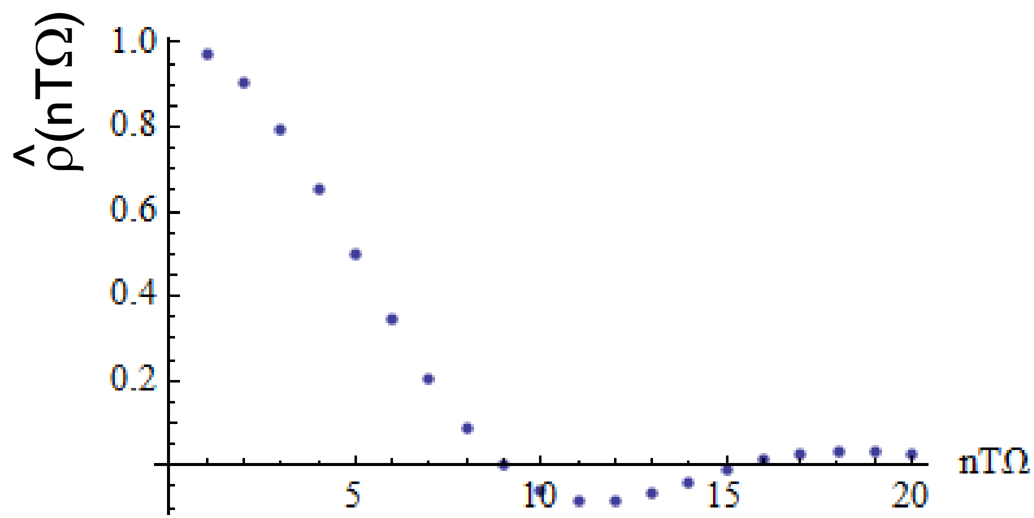
- Bret *et al.*, Phys. Rev. E **85**, 036402 (2012)

Example: 1D beam, parabolic time profile



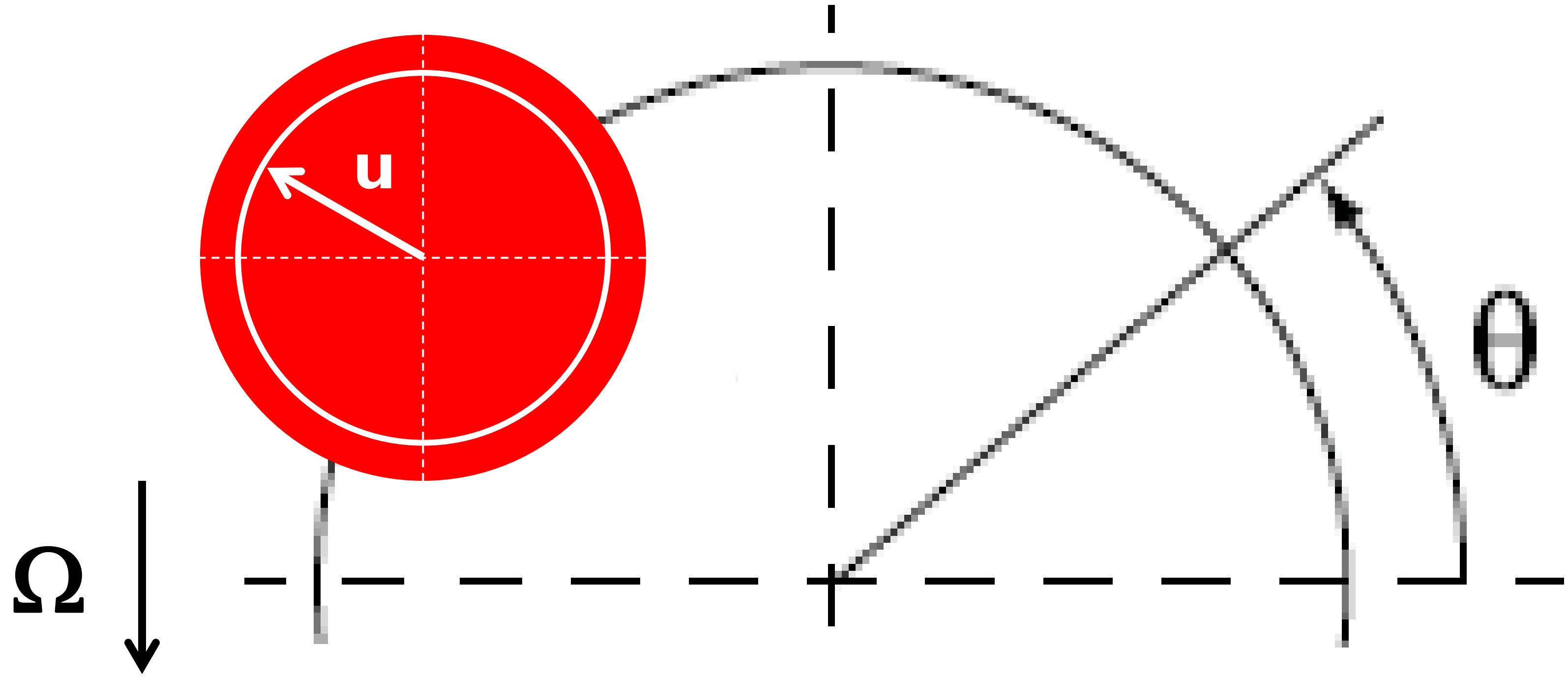






- Bret *et al.*, Phys. Rev. E **85**, 036402 (2012)

Irradiation Fourier spectrum, 2D beam



$$I(t,\mathbf{u}) = \xi(t)\sigma(\mathbf{u}) = \xi(t)\sigma(u)$$

Time profile

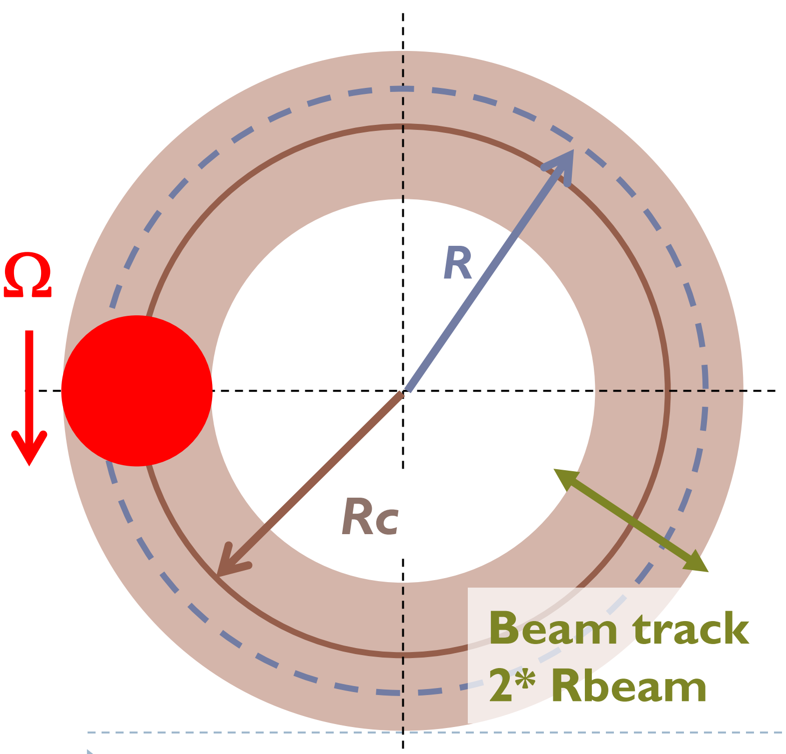
Beam transverse structure

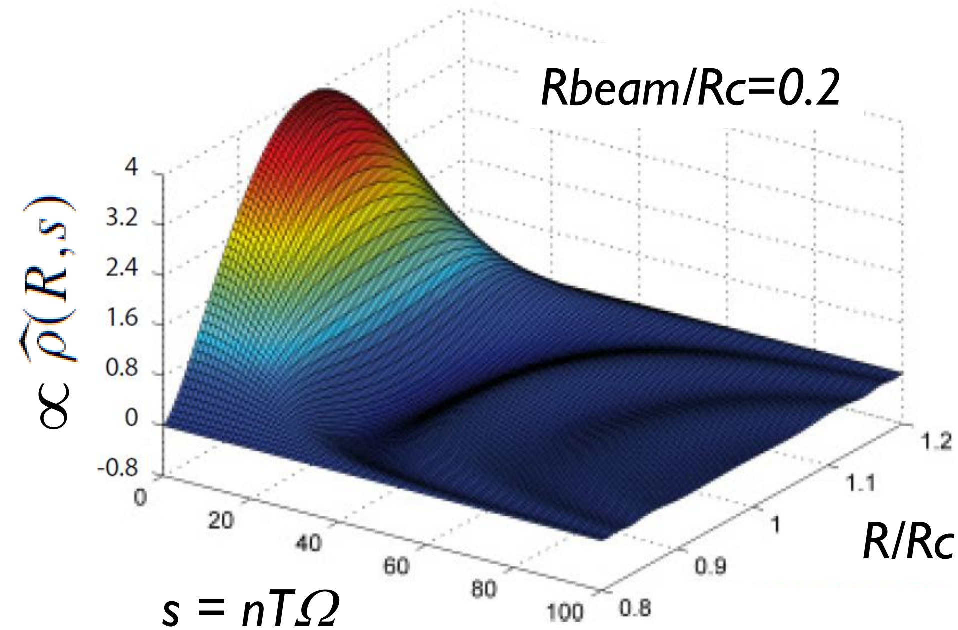
- Bret *et al.*, Phys. Rev. E **85**, 036402 (2012)

Irradiation Fourier spectrum, 2D beam

$$\underbrace{\frac{\widehat{\rho}(R,s)}{R\,dr} \equiv \widehat{\rho}_{2\mathrm{D}}(R,s)}_{\text{Irradiation harmonic at distance R}} = \underbrace{\left[\widehat{\xi}(s\Omega)\sum_{l=-\infty}^{\infty}\delta(s-l)\right]}_{\text{1D result}}\underbrace{\widehat{\sigma}\,[u(R,s)]}_{\text{Form factor}}$$

Rb << Rc
~ 1





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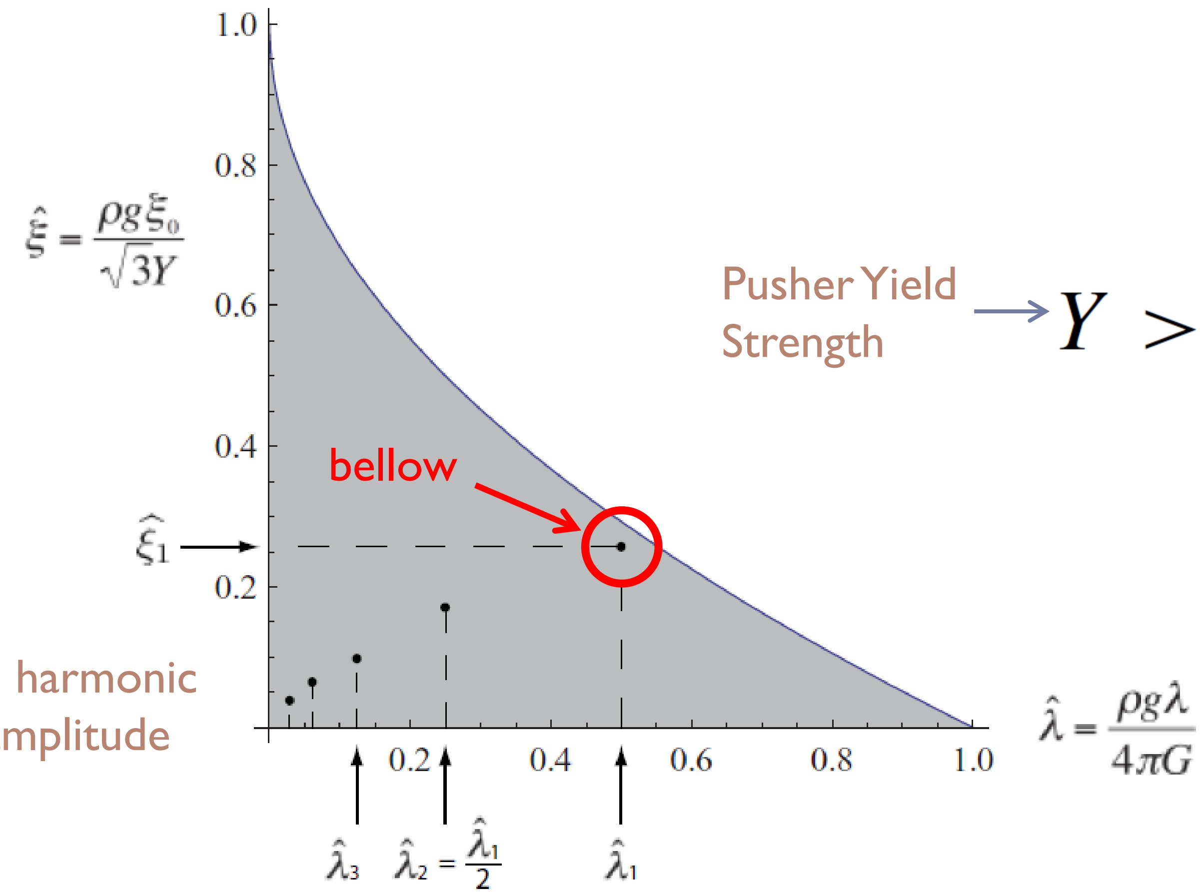
Stability analysis

- **"Stable if 1st harmonic is"**, gives back Piriz 2009



*n*th harmonic amplitude

Max. accelerating pressure

Pusher Yield Strength

$$Y > \frac{\sqrt{3}\, p_0}{\pi^2 N^2}$$

Nb of revolutions
$\Omega T = 2\pi N$

- Piriz *et al.*, Phys. Rev. E **80**, 046305 (2009)

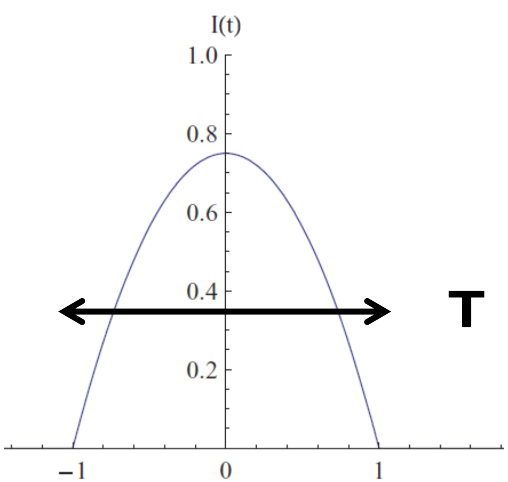
Bret *et al.*, Phys. Rev. E **85**, 036402 (2012)

Cancelling the 1st harmonic

- What about the planar approximation?
  - Piriz 2009 RTI-EPS analysis is in **planar** geometry
  - We are on a **circle**, radius *Rc*
  - Planar OK, for $\lambda << Rc$
  - Wavelength of n^{th} harmonic = Rc/n
  - What about $n = 1$, or 2 or 3?
- Harmonic amplitude ↓ with *n*.
  - In general, $n = 1$ is the largest amplitude
- Cancel the 1st harmonic? Possible
  - Smaller harmonics → **Better** symmetry
  - **Better** planar approximation. Only needed from $\lambda = Rc/2$

Cancelling the 1st harmonic

- The 1st harmonic reads H1= $\int_{-T/2}^{T/2} I(t)\cos(\Omega t)dt \xrightarrow[\Omega \to \infty]{} 0$
- When oscillating, ∞ number of Ω's cancel it.
- **Example:** 1D parabolic time profile



H$_1$=0

$$\frac{\Omega T}{2} = \tan\frac{\Omega T}{2}$$

$$\begin{aligned} \Omega T/2 &= 4.493, \\ &= 7.725, \\ &= 10.904, \\ &\vdots \\ &= (2k+1)\frac{\pi}{2} - \frac{1}{(2k+1)\pi/2} \end{aligned}$$

"Magic" ΩT's

$\forall$ k $\in$ **N**

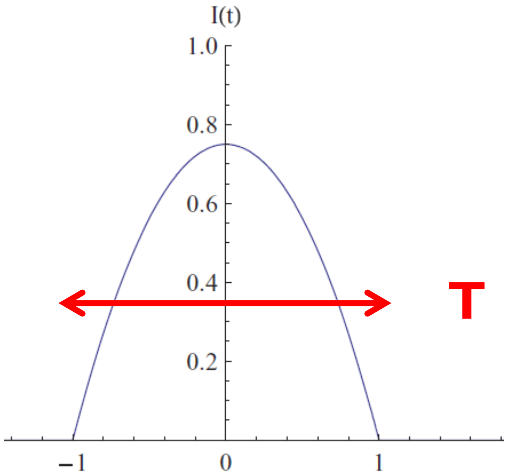
With these ΩT's, H1=0

- Bret *et al.*, Phys. Rev. E **85**, 036402 (2012)

Conclusions

- **RTI** in elastic-plastic solids is an issue for **LAPLAS**
- Growth % on wavelength & **amplitude** (Piriz 2009)
- Asymmetries excited by the **beam time profile**
- 1D beam: Harmonic amplitudes from the **Fourier transform** of the **beam time profile**
- 2D : Same, times a form factor ~ 1 for $R_{beam} << R_{deposition}$
- **Problem**: Planar RTI analysis dubious on circle for $\lambda = R$, + 1st harmonic too strong.
- **Cancel 1st harmonic**. Amplitude = oscillating integral.
- Infinite numbers of **"magic" ΩT's** work.
- Better **symmetry** + RTI analysis more **reliable**.

Cancelling the 1st harmonic



H1=0

$$\frac{\Omega T}{2} = \tan \frac{\Omega T}{2}$$